

# MORPHISMS BETWEEN QUOTIENT STACKS

LUCIEN HENNECART

ABSTRACT. When learning for the first time about quotient stacks, it is easy to think that any morphism between two quotient stacks comes from an equivariant morphism between the underlying algebraic varieties. Fortunately, it is very easy to get rid of this misconception.

Let  $G$  be an algebraic group (say over  $\mathbf{C}$ ). One may take  $G = \mathrm{GL}_n$  for concreteness. Let  $X$  be a complex algebraic variety. Then, the groupoid of morphisms  $X \rightarrow [\mathrm{pt}/G]$  is the groupoid of  $G$ -bundles over  $X$ . If there exists a nontrivial  $G$ -bundle over  $X$  (e.g.  $X = C$  is a smooth projective curve and  $G = \mathbf{G}_m$  is the multiplicative group, since there are nontrivial line bundles over  $C$ ), then the corresponding morphism  $X \rightarrow [\mathrm{pt}/G]$  does not come from an equivariant morphism  $X \rightarrow \mathrm{pt}$ . Indeed, there is a unique such morphism, corresponding to the trivial  $G$ -bundle over  $X$ .

SCHOOL OF MATHEMATICS, UNIVERSITY OF EDINBURGH, EDINBURGH, UK

*Email address:* `lucien.hennecart@ed.ac.uk`