Cohomological Study of Moduli Stacks

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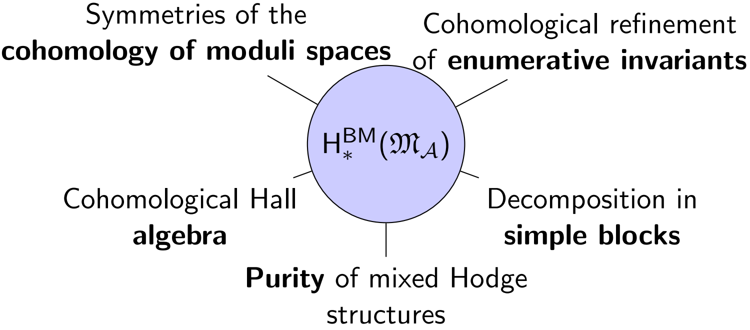
Topic: **Cohomological Donaldson–Thomas theory**

$\mathcal{A}$: Abelian category

$$\mathrm{Rep}(\text{Quiver}), \mathrm{Coh}(\text{Curve/Surf.}), \mathrm{Rep}(\pi_1(\text{Riem. surf.}))$$

$\mathfrak{M}_{\mathcal{A}}$: moduli stack of objects in $\mathcal{A}$.



Results:

- $\mathrm{H}^{\mathrm{BM}}_{*}(\mathfrak{M}_{\mathcal{A}}, \mathbb{Q}) \overset{\text{v.spaces}}{\cong} \mathrm{Sym}\left(\mathrm{BPS}_{\mathcal{A}} \otimes_{\mathbb{Q}} \mathbb{Q}[u]\right)$
- $\mathrm{BPS}_{\mathcal{A}} \overset{\text{Lie alg.}}{\cong} \mathfrak{n}^{+}_{\mathcal{A}}$ positive part of a generalised **Kac–Moody Lie algebra**.

Applications:

- **Positivity** of some counting polynomials over finite fields
- **Action of Lie algebras** on the cohomology of moduli spaces
- Progress in **nonabelian Hodge theory** for stacks

Future directions:

- Understand $\mathrm{H}^{\mathrm{BM}}_{*}(\mathfrak{M}_{\mathcal{A}})$ as an **algebra**
- Study $\mathrm{H}^{\mathrm{BM}}_{*}(\mathfrak{M})$ when $\mathfrak{M}$ is a 0-shifted symplectic stack
- Study $\mathrm{H}^{*}_{\mathrm{crit}}(\mathfrak{M})$ when $\mathfrak{M}$ is a (-1)-shifted symplectic stack